Homework 2

First Level

1)

We can use the combination formula to solve this problem. The number of ways to choose 4 students out of 12 for the first test is:

C(12,4) = 495

After these 4 students have taken the first test, there are only 8 students left to choose from for the second test. The number of ways to choose 4 students out of 8 for the second test is:

C(8,4) = 70

Finally, after 8 students have taken the first two tests, there are only 4 students left to choose from for the third test. The number of ways to choose 4 students out of 4 for the third test is:

C(4,4) = 1

To find the total number of ways that the tests can be taken, we multiply these three numbers together:

495 x 70 x 1 = 34,650

2)

2\*3\*1=6

we can see all six permutations: abc, acb, bac, bca, cab, and cba

3)

A) The total number of ways to select two items from a box containing 12 items is:

n = 12C2 = (12! / (2! \* 10!)) = 66

Let A be the event that both items are defective. There are 4 defective items in the box, so the number of ways to select two defective items is:

m(A) = 4C2 = (4! / (2! \* 2!)) = 6

Therefore, the probability of selecting two defective items is:

P(A) = m(A) / n = 6 / 66 = 1/11

Let B be the event that both items are non-defective. There are 8 non-defective items in the box, so the number of ways to select two non-defective items is:

m(B) = 8C2 = (8! / (2! \* 6!)) = 28

Therefore, the probability of selecting two non-defective items is:

P(B) = m(B) / n = 28 / 66 = 14/33

Therefore, P(A) = 1/11 and P(B) = 14/33.

B) P(at least one item is defective) = 1 - P(no item is defective)

To find P(no item is defective), we need to count the number of ways to select two non-defective items from the box. We know that there are 8 non-defective items in the box, so the number of ways to select two non-defective items is:

m = 8C2 = (8! / (2! \* 6!)) = 28

Therefore, the probability of selecting two non-defective items is:

P(no item is defective) = m / n = 28 / 66

Substituting this value into the formula above, we get:

P(at least one item is defective) = 1 - P(no item is defective) = 1 - (28 / 66) = 38/66

the probability that at least one item is defective is 38/66.

4)

We can use the formula for probability:

P(event) = (number of favorable outcomes)/(total number of possible outcomes)

(i) To find the probability that none of the three selected items is defective, we need to choose 3 items from the 10 non-defective ones. The total number of possible outcomes is choosing 3 items from all 15. Therefore,

P(none defective) = (number of ways to choose 3 non-defective items)/(total number of ways to choose 3 items)

= (C(10,3))/(C(15,3))

= (120)/(455)= 24/91 =0.2637

(ii) To find the probability that exactly one item of the three items is defective, we need to choose one defective item and two non-defective ones. There are C(5,1) ways to choose one defective item and C(10,2) ways to choose two non-defective ones. Therefore,

P(exactly one defective) = (number of ways to choose 1 defective and 2 non-defective items)/(total number of ways to choose 3 items)

= (C(5,1)\*C(10,2))/(C(15,3))

= (5\*45)/(455)

= 45/91 =0.4945

(iii) To find the probability that at least one item of the three items is defective, we can use the complement rule: P(at least one defective) = 1 - P(none defective). Therefore,

P(at least one defective) = 1 - P(none defective)

= 1 - (C(10,3))/(C(15,3))

= 1 - (120)/(455)

= 67/91 = 0.7363

5)

P(boy) = number of boys / total number of students = 10 / 30 = 1/3

The probability of choosing someone from Mansoura randomly is:

P(Mansoura) = number of Mansoura students / total number of students = (5+15) / 30 = 2/3

P(boy or Mansoura) = P(boy) + P(Mansoura) - P(boy and Mansoura)

= (1/3) + (2/3) - (5/30)

= 20/30

= 2/3

Therefore, the probability that a person chosen randomly is a boy or from Mansoura university is 2/3.

6)

(i) P(Ac) = 1 - P(A) = 1 - 3/8 = 5/8

(ii) P(Bc) = 1 - P(B) = 1 - 1/2 = 1/2

(iii) P(Ac intersection Bc) = P((A union B)c) = 1 - P(A union B)

= 1 - (P(A) + P(B) - P(A intersection B))

= 1 - (3/8 + 1/2 - 1/2)

= 5/8 - 1/4

= 3/8

(iv) P(Ac union Bc) = P((A intersection B)c)

= 1 - P(A intersection B)

= 1 - 1/2

= 1/2

(v) P(A intersection Bc) = P(Bc|A)\*P(A)

=(P(B)-P(B intersection A))/P(A)\*P(A)

=(1/2-1/2)/3/8

=0

(vi) Similarly, we can find that

P(B intersection Ac)=0

7)

The complement of rolling at least one 7 is rolling no 7s in three rolls. The probability of this happening is (5/6)^3 = 125/216.

Therefore, the probability of rolling at least one 7 in three rolls is:1 - (125/216) = 91/216

8)

The sum of probabilities of all possible outcomes in a sample space is always equal to 1. Therefore:

Σ P(x) = 1

However, in this case, we are given that:

Σ P(x) = k^2 - 8

Therefore:

k^2 - 8 = 1

Solving for k, we get:

k^2 = 9

k = ±3

Therefore, the value of k is ±3.

9)

P(A′ ∩ B′) represents the probability of outcomes that are not in A and not in B. This can be calculated as follows:

P(A′ ∩ B′) = 1 - P(A ∪ B)

Since A and B are mutually exclusive, their union is simply the sum of their probabilities:

P(A ∪ B) = P(A) + P(B) = 0.35 + 0.45 = 0.8

Therefore,

P(A′ ∩ B′) = 1 - P(A ∪ B)

= 1 - 0.8

= 0.2